

Papy's Minicomputer

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The method we have used to introduce children to mechanical and mental arithmetic employs the distinct advantages of the binary system over other positional systems, while at the same time taking account of the decimal environment in which we are immersed. It is thanks to the *Mini-computer Papy* that we have been able to achieve this.

Inspired by some work of Lemaitre, this variety of two-dimensional abacus uses the binary system on boards which are arranged according to the decimal system.

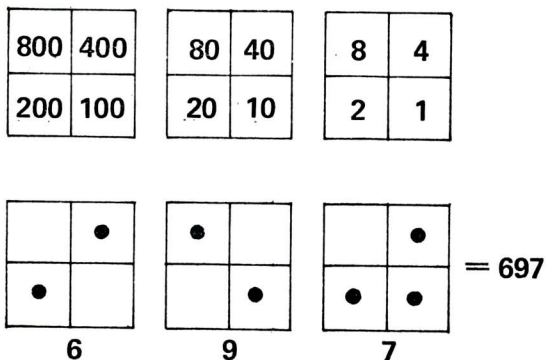


Fig. 1

The colours recall the red family of the set of Cuisenaire rods and help to make the four rules of the machine easily accessible.

2nd October, 1967

Frédérique hangs one of the display boards on the blackboard

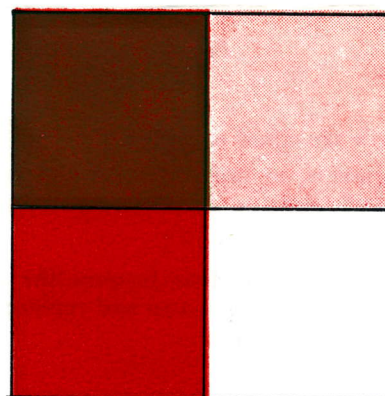


Fig. 2

- Oh, they're our colours! said Jean-Jacques, showing a white, red, pink and brown rod.
- Yes, indeed!
- We will play a game together; you will use your rods and I will use the board and these counters.
- Frédérique puts two black counters on the white square.
- They stay up by themselves! one child marvelled.
- Couple two white coaches together.
- It's the same as the red rod!
- A white-white train is the same length as a red coach.
- Frédérique takes two counters from the white square and puts one on the red square,

5th October, 1967

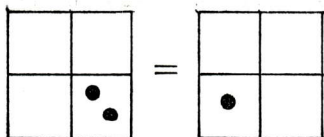


Fig. 3

- Two counters on the white square are equal to one on the red.

The children are quick to adopt the shorthand 'a red' for 'a counter on the red square' and state the rule 'two whites are equal to a red'.

- Let us go on with the game!

Frédérique puts two counters on the red square and the children put two red rods together.

- It is the same as the pink rod!

Frédérique takes the two counters from the red square and puts one on the pink square.

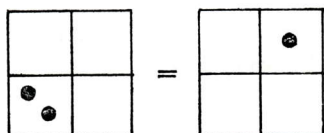


Fig. 4

- Two reds are equal to a pink.

Jean-Jacques is on fire with desire to move the counters and begs:

- Madame! Can I play?

Frédérique agrees and Jean-Jacques lifts the black counter from the pink square and replaces it with two green counters.

- It is beautiful!

- Couple two pink coaches together.

- It is the same as a brown rod.

Frédérique removes the green counters and puts one on the brown square.

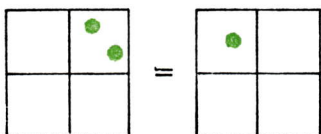


Fig. 5

- Two pinks are equal to a brown.

- A new game!

Frédérique places a counter on the white square

- 1

She places a second counter on the white square

- 2

- I can play another way! says Carine. She lifts off the 2 counters from the white square and puts one on the red.

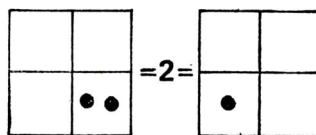


Fig. 6

- Add 1 to the number 2, Frédéric suggests, putting a new counter on the white square.

- It is 3.

- Like with rods, red-white

- or light green.

- Add 1 to the number 3, Frédéric continues, putting a new counter on the white square.

- It is 4.

- I can play another way, says Sylvie, who takes the two counters from the white square and puts one on the red.

- And another way, states Jean-Jacques, replacing the two counters on red by one on the pink.

- Let us go on adding 1 . . .

The recital continues and the numbers 5, 6, 7, 8, 9 appear on the Minicomputer.

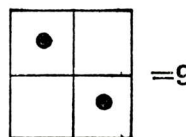


Fig. 7

- Add 1 to the number 9.

A child puts a counter on the white square.

- It is 10.

- I play in another way, says Anita, replacing the two counters on white by one counter on the red.

- It's the red-brown train.

- The orange rod.

- It's 10.

Frédérique hangs a second Minicomputer board on the blackboard, to the left of the first. She takes the two counters from the red and brown squares and puts a counter on the white square of the new board, saying simply

- And this is still 10.

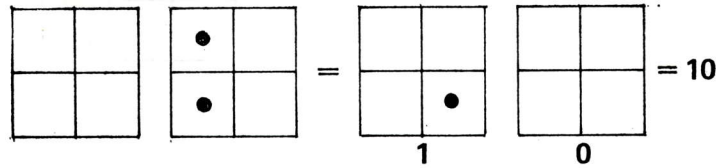


Fig. 8

- Jump! On to the second board, remarks Jean-Jacques, not in the least surprised.

24th October, 1967

Thus armed, our pupils are able to represent on the Minicomputer the number of elements in any set of counters put on square 1. The application (in reverse order) of the four fundamental rules on a small scale lets us keep a concrete link between a number in its written form and as represented on the machine.

13th October, 1967

Two small Minicomputer boards and a box of counters are on the desks in front of each child.

- How many counters do you have in your box?
- 29 ... 32 ... 18 ... 26 ... 35

The children's boxes are not all filled with the same number!

- Put the box of counters on the white square of the first board. Play ... and then write down the result.

The first extended piece of individual work on the Minicomputer: concentration, precise and quick movements: some children arrive at the result without a mistake.

Clumsy techniques, counters spilled, false moves; the others must be helped.

The second part of the lesson is played on the wall Minicomputer.

We start with the number 25 on the machine. We work all the counters back to the white square on the first board and count them: confirmation!

After a fortnight the demands of the class forced Frédérique to introduce a third board and to accept numbers over 100.

- If I put a counter on each square of the machine, will I make the number 100? asks Didier, who still has a machine with only two boards.

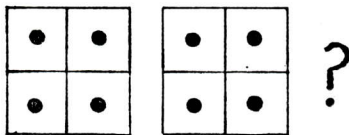


Fig. 9

Frédérique does not answer. She expects the problem to arise later in a different form and prefers to let the child's thinking follow its own course.

Didier returns to the attack.

- I want to make 100 on the machine.
- All right ... let us have your method!
- 100 is twice 50, Didier goes on, showing:

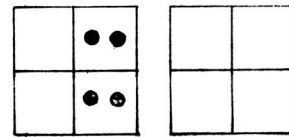


Fig. 10

- I can play!

He replaces the two counters on the white square by one counter on the red square and the two counters on the pink square by one on the brown.

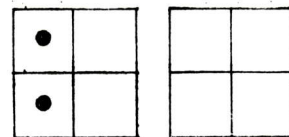


Fig. 11

- I want to play again! he says, taking the counters from the red and brown squares and putting a counter on the left of the second board.

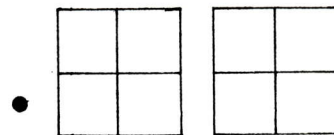


Fig. 12

And Didier demands his rights:

- I must have another board, Madame!
Frédérique gives him one. Didier triumphantly forms the number.

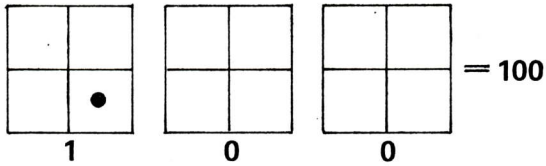


Fig. 13

Overexcited, he shouts out the numbers as he shows them on the machine.

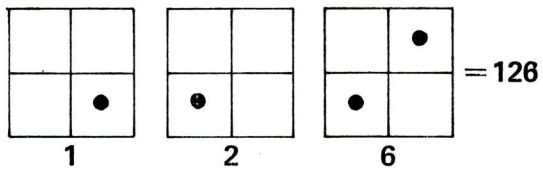
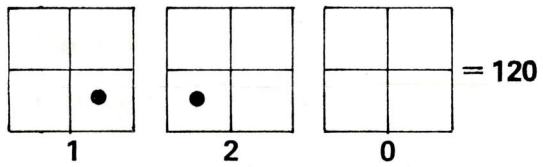
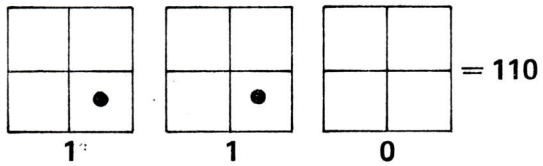
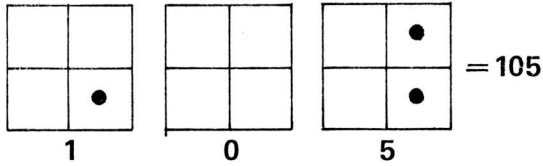


Fig. 14

Impressed, the class has shared in this discovery.

The addition of whole numbers is carried out automatically by 'playing' the machine. The same is true for *doubling* which is a primitive experience for children and fundamental to the Mini-computer.

The part played by arbitrary memorisations, so often a distasteful part of learning to compute, is reduced to a minimum. The addition of small numbers is carried out by means of intelligible rules: a purely binary system when the sum is less than 9 and a mixed decimal-binary system in other cases. From the beginning, therefore, the children are initiated into a positional system of numeration,

- In the car park I counted 75 Volkswagens and 49 Mercedes. How many cars are there altogether?
- Show 75 in red!

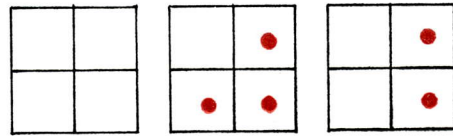


Fig. 15

- And 49 in green!

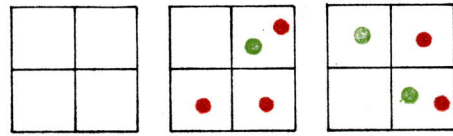


Fig. 16

- Can I play, Madame?

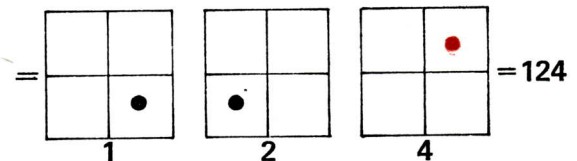
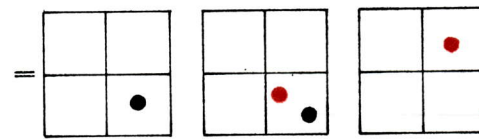
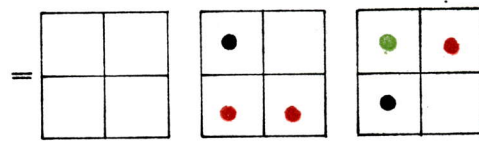
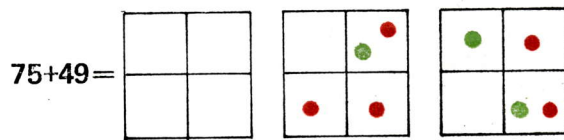


Fig. 17

The children make a note of the temperature in degrees centigrade (Celsius) each morning. At the start of the school year, in Brussels, these are always natural numbers, but the winter months force the use of negative numbers. From the *sixth month*, negative integers are part of the common knowledge of the children, having authentic status as numbers 'measuring' a 'size' which has been experienced.

The results of a sequence of two-person games are written with red and blue numbers which are mutually destructive, since each point captured by one of the players 'kills' a point captured by the other. The addition of red and blue numbers follows. From the given notation the children arrive at the additive group of integers.

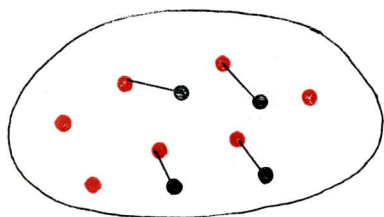


Fig. 18

Eventually, the writing is simplified and all the numbers are written in black, those originally blue having a bar across the top. In effect this time we have the group $Z, +$, except for a very minor difference, $\bar{3}$ being used instead of -3 . We have known the advantages of the $\bar{3}$ notation for beginners for a long time since we have used it to simplify logarithmic calculations.

At the beginning we show the result of each game by putting red and blue counters on a flat surface. A battle to the death gives the final score. The children spontaneously transfer the procedure to the Minicomputer.

- Let us calculate this to find how big it is:

$$100 + 23$$

Great excitement at all the desks!

- Do it on the machine!

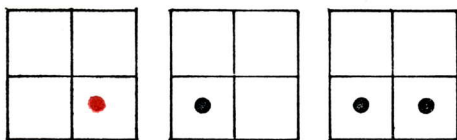
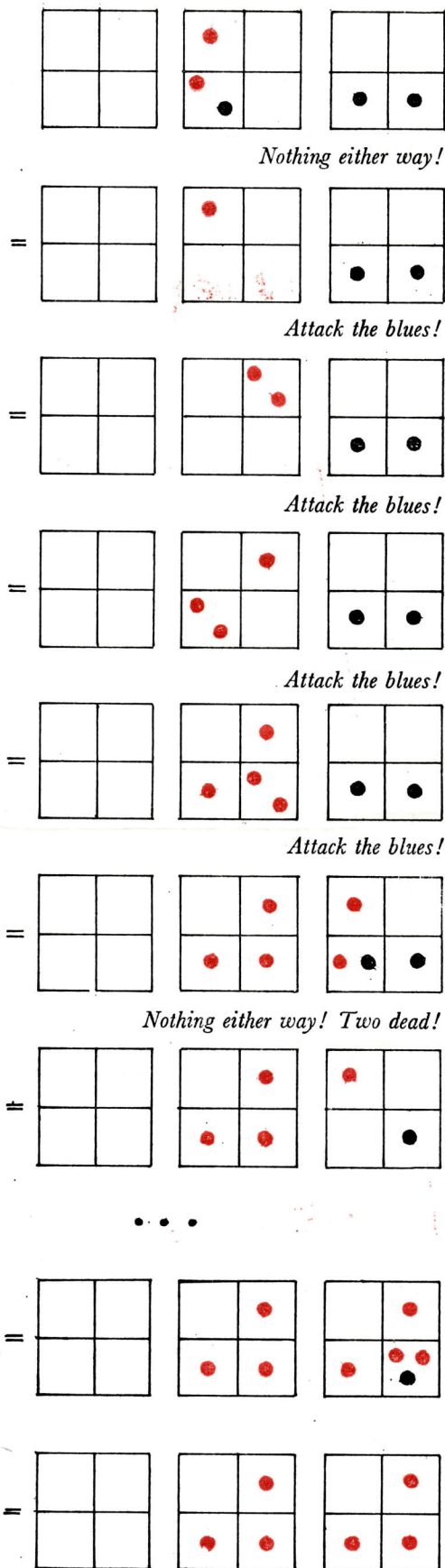


Fig. 19

□ The black dots in Figs. 18, 19 and 20 should be interpreted as being blue.

Fig. 20 (opposite)



$$100 + 23 = 77$$

- Who will win?
- Red!
- Red soldiers! Attack the blues!

In our teaching, the function 'a half of' appears as the reciprocal of the function 'twice' which is a transformation of \mathbf{Z} ; that is, a function mapping \mathbf{Z} into \mathbf{Z} . But some numbers exist which are not twice an integer. They do not have integral halves.

A bar of chocolate can be broken fairly into two pieces. A 100 franc note can be changed into two 50 franc notes. 100 is certainly an even number. But one franc can be changed into two 50 centime coins, and fortunately this always appeals to six-year-old children. Consequently it seems very natural to them to try to find a half of 1 on the machine.

It was because they wanted to write 100, got by doubling 50, that the pupils literally obliged me to give them the third board. We can now put 100 on the board and watch the way in which we find a half of it; we can start with 10 in the same way.

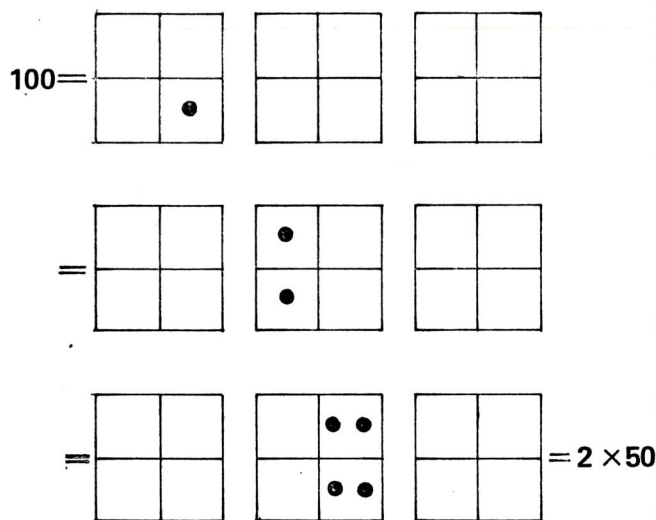


Fig. 21

How can the same rule be applied to finding a half of 1? The children ask for a new board, to the right of the others, and at once call it the 'tiny numbers board'. How shall we remember that it is the 'tiny numbers board'? By putting a green line between the boards which will later become the decimal point. So we can calculate a half of 1 and we write 0.5.

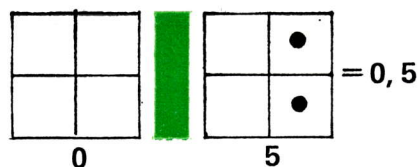


Fig. 22

The problem of dividing 100 francs fairly between three children introduces an exceptionally interesting situation. The first glimpse of a non-terminating decimal comes through in this remark of one child: 'We will still be here tomorrow morning...'

The interest which six-year-old children show in quite large numbers forces us to respond by giving them unmotivated calculations which offer various kinds of challenge. Thanks to the Minicomputer, our pupils can add and subtract three-figure numbers and multiply them by simple fractions. The work is a good foundation because it requires considerable concentration to find the best strategies each time. These exercises help to produce a harmonious cooperation between the intelligent human being and a true machine, which is what the Minicomputer is in the eyes of the pupils.

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